1. INTRODUCTION

In the recent ten years prices of key commodities, particularly the non-iron metals were strongly varying and hardly predictable [1, 2]. Partially, it was caused by noticeable changes in a position of UD dollar in World economy, which could affect in a similar way exchange rates of more important currencies [3, 4, 5]. Hence, one may expect that a new properly defined currency could be used to decrease the prices variability, and so to reduce the risk of transactions in World commodity markets.

Recently, some media outlets started to discuss the introduction of a global currency, which in time would replace not only the dollar, euro and British pound, but also currencies of other countries in the world. Many well-known economists think that the global crisis is the best chance for a global currency [6, 7]. Enthusiasts of a global currency use arguments mainly based on the current economic crisis. The crisis has caused among others a sharp decline in international trade and a weakening U.S. dollar weakened foreign currency reserves of many countries in the world. The costs of trade settled in the dollar have increased, the most obvious example being a rise in oil prices, and with it the costs of fuel. Combating the effects of the global crisis was discussed during the G20 meeting held in April in London. Dissatisfied with the depreciating dollar is China, which invested most of its reserves in that currency. It began to call for the replacement of the dollar with a new global currency, a function which could be fulfilled by special drawing rights (SDRs are also called international paper money. They are international reserve assets, created by the International Monetary Fund in 1969 and used as the unit of account. They are an instrument to shape foreign exchange reserves.)

The primary objective in the creation of SDRs was to avoid Tryffin's dilemma in the Bretton Woods monetary system, the opposition between the international character of use and the national nature of currency. One of the main problems that SDRs should resolve is the use of a new means of settlement as a basis of the international monetary and credit mechanism, the transition from the gold-standard to the SDR standard. It was assumed that SDRs would be quality alternatives to gold, U.S. dollars and other currencies functioning as foreign currency reserves and means of payment. Such a collective unit of currency should work as a stabilizer by reducing the consequences of breaches of the balance of payments and be a barrier and a brake on the widening disparities that result from violations of balance.
Can the global confidence crisis be cured by virtual currency? SDRs are in fact merely a unit of account for IMF member states, the IMF. They are not backed by guarantees in the form of assets (obligations, shares or other). Experts, however, warn that you can not say that it is money without cover. - It is money based on mutual trust. It will function until there is confidence in the IMF member states. In addition, SDR units are fully convertible into other currencies between the central banks of member states. However, financial institutions are already seeking to have the right to issue them as a settlement unit between banks. Currently, there is no talk about a generalized introduction of SDRs into circulation.

2. INTERNATIONAL MONETARY POLICY – IMF and SDR

2.1. International Monetary Fund

The International Monetary Fund (IMF) began operating in 1945, when 29 countries, controlling sixty-five percent of the total shares in the Fund, ratified on the 22 July 1944 at the Bretton Woods allies' conference the Articles of Agreement of the International Monetary Fund (Article XX of the Bretton Woods Agreement). The IMF, in conjunction with the International Bank for Reconstruction and Development, was designed to carry out a permanent supervision of the stability of international monetary affairs, as well as boosting the development of world trade. IMF is oriented towards the continuous monitoring of exchange rates, the provision of short-term financing as well as technical measures to enable the proper functioning of member states' currencies. The main objective of the Fund is to create the conditions necessary for financial and economic stability in member countries, maintenance of monetary stability, creating frameworks to facilitate and promote the exchange of goods, services and capital, and promote economic growth (Zabielski, [8]).

The main functions of the Fund include the function of regulatory and financial advisory (Bilski, [9]). Member States are participants in the IMF's founding conference in Bretton Woods. Successive members are acceding to the conditions and time limits set by the organization. It currently has 185 states, and the last, which joined its ranks on 18 January 2007, is Montenegro. The IMF's two most important advisory bodies are the International Monetary and Financial Committee of the Board of Governors' which focuses on the global financial situation with the aim to adapting the international monetary system, to ensure the currency liquidity and the elimination of potential risks. The Development Committee on the other hand focuses on the transfer of resources to developing countries. The Fund also deals with a multitude of departments, regional, functional and other.

According to the IMF, the main economic problems in the world are due to the improper currency balance between countries. The classic example of such a problem is the rapid depreciation of the currency of one country, which automatically results in an increase in prices of foreign goods and capital which ultimately leads to disruption of the economy as a whole (the domino effect). Countering such phenomena, the Fund supports Member States in implementing reforms and adjustment programs, creating favorable conditions for sustainable economic growth, stability of employment and improving investment dynamics. Loans are therefore granted for specific purposes, whose implementation will result, directly or indirectly,
in improving the economic capability of the beneficiary, and thus debt repayment is almost guaranteed.

More than 60 years after the Bretton Woods conference, the importance of IMF's role in the development of the global economy is undeniable. However, because of changes in the international situation it is necessary to continuously adapt strategy to challenges such as the rapid development of emerging markets and the growing tendency among some countries to unilateralism (www.imf.org/external/np/speeches/2006/041006.htm). The IMF as the world's main monetary institution will continue to play a major role as global regulator and supervisor of the global monetary system, regardless of the specific shape of that system (Lutkowski, [10]).

2.2. The SDR Phenomenon

Since the beginning, special drawing rights were intended to support the new exchange-rate system of Bretton Woods. Countries belonging to this system were required to have official reserves or central bank reserves in the form of gold and foreign currencies, which could be used to purchase the national currency on world foreign exchange markets in order to maintain its exchange rate. However, international demand for two key units of reserves, gold and the U.S. dollar has slowed the expansion of world trade and financial development - hence the decision by the international community to create a new global reserve unit under the auspices of the IMF. This unit was experimental, because it did not have, like national currencies, security in the form of support provided by the productive potential of a particular country. There were great concerns that SDRs may be used as a tool to change the structure of the reserves. Seeing this opportunity authors of the SDR settlement concept added extra security. Two rules were adopted: the first said that in a situation where the country would use up its allocation of SDRs, it would be required to purchase these units back, so that the annual average balance would not descended below a minimum 30% of the original allocation. The second principle laid down the conditions under which one could buy the currency of another country with SDRs, and for what purposes it can be used. This option was possible only with the consent of the IMF, who designated the members from which currency could be purchased. The purchased currency had to be used for genuine commercial and capital transactions, and not for changing the structure of reserves (Lutkowski, [10]).

Currently, the use of SDR is not encumbered with protective regulations and transactions take place freely. Only the purchasing of currency for SDRs is controlled. SDRs can be exchanges for currencies in two ways: the first allows for voluntary exchanges between members, the second involves "designated" members of the IMF with strong external positions being allowed to purchase SDRs from members with weak external positions.

SDRs function since 1970, when the International Monetary Fund made its first issue. In the years 1970-1972 and 1979-1981 the total subscription was 21.4 billion SDRs. As a result, at the end of 1995 they accounted for 1.6% of foreign exchange reserves (including gold), and at the end of 1999 1.2% (Zabielski, [8]). The IMF took the decision to further issue SDR 21.4 billion in 1997. The SDR units were allocated to countries which have not yet received SDRs. The third issue occurred in 2009 and amounted to the sum of 161.2 billion SDR (equivalent to
250 billion U.S. dollars). In addition, 21.4 billion SDR (33 billion U.S. dollars) were to be allocated to countries that previously did not belong to the IMF. This was decided at the G20 Summit in April 2009. It was to be one of the means to halt the crisis. Was it realistic? - If these units reach the deficitary countries, this will undoubtedly ease payment difficulties and support the growth of international trade. Indeed, if reserves are depleted, a reduction in foreign currency expenditures will follow which creates a barrier that harms all members of the IMF. The value of SDRs was initially fixed to parity with 0.888671 g of gold, which was the equivalent of one U.S. dollar (U.S. 1 SDR = 1 dollar). After the collapse of the Bretton Woods system of 1973, the value of SDRs was based on a basket of currencies.

Currency baskets are a system for establishing the value of the SDR international monetary unit. Starting July 1, 1974 it was established that the SDR will be based on a basket of 16 currencies, each of which had to account for no less than 1% of world trade, and weighed according to economic turnover and size of currency reserves of the given member country. From 1 January 1981 the SDR basket was based on five currencies: the U.S. dollar, the German Mark, French Franc, Sterling and the Yen. In 1986, 1991 and 1996 adjustments were made to the weight of currencies in the basket resulting from changes in the currency markets. In connection with the introduction of the euro, the basket was revised again. This took place in 1999 when the German Mark and French Franc were replaced by the German and French Euro. Since 2001, the basket consists of 4 currencies: the euro (34%), the U.S. dollar (44%), yen (11%), and pound sterling (11%). Determination of the weights in the basket is based on the level of trade and the level of reserves of the countries included in the basket. These weights are updated at 5 year intervals. The basket is also used to calculate the SDR interest rate. The interest rate depends on the exchange rate basket. It is calculated according to the formula: currency share in basket * currency value in relation to the SDR * corresponding interest rate = interest rate involved in the SDR. (E)

An important advantage SDRs is the fact that their value is specified by the basket four important (perhaps most important) currencies and it is they who designate the "SDR rate." Thanks to this the SDR is more stable because the decline in the value of one currency is compensated by the increase in the others. Thus, a depreciation of the dollar is usually observed in conjunction with the appreciation of the euro or other currency in the basket. Central banks are thus interested in SDRs since they helps to avoid losses due to sudden depreciation in reserve currencies. The value of currencies composing the SDR is communicated every day on the IMF website. The SDR interest rate is fixed each week on the basis of a weighted average of interest rates on short-term bonds of the currencies composing SDRs. Some banks offer investment deposits based on a basket of currencies. (For example, in Poland, the BGZ Bank offers its clients an investment based on the four currencies included in the currency basket: EUR / PLN, EUR / CZK, USD / TRY and USD / RUB. The share of each of the four currencies is 25%). The use of the SDR basket is also considered by the OPEC countries. This is mainly due to large fluctuations in the dollar, from which OPEC wishes to dissociate oil prices.

Countries belonging to the IMF may use SDRs as new sources of liquidity, which is to compensate for deficits in the balance of payments. They may also provide loans, fulfill their obligations and give donations in SDRs. States with surpluses in excess of their allocation of
SDRs obtain interest payments on them, while countries in SDR deficits are obliged to pay interest. Settlement of the percentages is made quarterly. IMF does not have the mandate necessary to issue and manage SDRs. All decisions regarding the issue of new SDRs are taken by the governments of member countries. Therefore, changing the composition of the currency basket is likely to be associated with long and complicated decision-making process. Thus, a financial policy aimed at stabilizing prices of new SDRs seems to be unrealistic. Also, it is doubtful whether all the countries will be satisfied by the umbrella of the single currency. Perhaps it would be best to return to the equivalent of the gold standard. Central banks store a large amount of gold as reserves (the G20 central banks have two thirds of global gold reserves), but even this quantity of gold is not large enough to cover all the paper money in the world without destabilizing prices of gold. Perhaps the basket should contain several kinds of raw commodities (long ago, such a view has been expressed by the British economist John Maynard Keynes at the Bretton Woods conference, where he proposed a currency called the "bancor". According to him, the basket should contain 30 different commodities). It should be noted that the attractiveness of gold and other commodities is that they present central banks with rather hard requirements, among which the most important may include control over monetary policy. If this were the case, speculative bubbles and jumps in inflation would be impossible. The danger of protectionist competition (devaluation) would also disappear because of the strong currency link to gold.

2.3. The Situation of SDR Basket Currencies

In March 2009, during the London G20 summit, world leaders wondered how to rebuild the credibility of the financial system and how to get out unscathed from the global crisis. The Americans and the British proposed pumping billions of dollars to boost consumption and investment to spur economic growth. Germany and France more cautiously stressed attention to growing budget deficits. British Prime Minister Gordon Brown and Brazilian president Luiz Inácio Lula da Silva called for the creation of a 100 billion dollars fund aimed to enliven world trade. At issue was the regulation of financial markets. G20 finance ministers agreed, inter alia, about greater control over hedge funds and rating agencies. But among world leaders there was disagreement as to how much the state should intervene in market activity.

G20 leaders agreed to strengthen the International Monetary Fund. (Even before the meeting, several countries that fell into financial trouble, have received assistance from the IMF). Fund authorities announced that further support they may be impossible for the lack of money. Therefore, representatives of the Fund and the EU pushed for a doubling of the IMF funds to over 500 billion dollars. World powers - Russia and China have argued for the need for changes in international currency. The head of the Chinese central bank Zhou Xiaochuan urged the IMF to increase the role of the SDR - the currency of account of the IMF, whose value depends on the currency basket. He also called for the creation of a global currency reserve. The Vice President of China Construction Bank added that such proposals have in fact already undermined the credibility of the dollar. IMF director Dominique Strauss-Kahn said that the proposed introduction of a new reserve currency in place of the U.S. dollar is justified, and that talks on this issue would begin in the coming months.
At the Economic Forum in St. Petersburg in June 2009, Russian Finance Minister Kudrin assessing the activity of the IMF pointed out that it can play the role of locomotive of the world economy, provided that it is thoroughly reformed, and above all, that the structure of the IMF will reflect the economic strength of member states. Meanwhile, several small European countries have a greater representation in the Fund than China. The Chinese, who have located in U.S. bonds more than one trillion U.S. dollars, proposed solutions to emancipate SDRs from the American currency. Russia proposed that the SDR exchange rate were based on a basket including the ruble, Yuan and gold. In connection to this, many well-known figures in the financial world discussed and pointed to the need for reforming the IMF. Also, the IMF Director General Dominique Strauss-Kahn in his speech on 18 November 2009 indicated that the global community can rely on a uni-polar system which was formulated after the departure from gold system. He maintained that there will probably be a need to depart from the reserve currency in favor of a basket of currencies. SDRs can be used as the basis of the currency basket. He stressed that this reform should be carried out during the current year. A similar opinion was expressed also by the former head of the IMF Michel Camdessus, who claimed that now is the best time to reform the global monetary system and that a similar opportunity will not repeat soon. At the same time he reiterated the proposal of the National Bank of China to expand the SDR basket. Maybe then the SDR can replace the dollar and become a real reserve currency. For this purpose, SDR basket should be transformed so as to include also the large developing world currencies, the Chinese Yuan, Indian rupee, and the Brazilian Real.

Indeed, with the advent of the world financial crisis the problem of a new global reserve currency becomes extremely urgent. In the past, this issue was rarely raised, however. Currently, the economics of the situation has changed so that the discussion about an alternative to the dollar as reserve currency has gained momentum. Referring to this problem, EU Commissioner Benita Ferrero-Waldner said that at present it is too early to consider the proposals of China and Russia. But the problem is not only a diplomatic problem, as the rising debt, budget deficits of countries and the lack of liquidity undermined confidence in the financial system based on the dollar. In this situation, continuous use of the dollar as a reserve currency without a closer analysis may lead to new crises. U.S. President Barrack Obama, as well as the representative of the Fed Ben Bernanke say that today there is no alternative to the dollar, which is currently strong as ever and the U.S. is coming out of the crisis. Secretary of Treasury Timothy Geithner said that while the U.S. is certainly open to China's suggestions, the dollar will remain the leading reserve currency for a long time still. At the same time a huge external US debt and budget deficit worries economists. The accumulated debt and budget deficit reaches 12% of GDP. (Paradoxically, difficult economic situation and the risk of insolvency often will foster substantial economic reforms). However, the belief in the healing influence of the crisis should be moderated. At the same time the central banks of the world use U.S. dollars for funding purposes. The share of dollar reserves is 65% of global currency reserves. In view of this fact, the dollar has a considerable advantage, and the situation stimulates demand for the dollar. Such a favorable situation for the United States has been continuing for decades. However, you can cautiously say that recently the U.S. has started to ignore the status of the dollar, using it to prop its weakening debt-based economy. Therefore one could get the impression that the status of the dollar as currency reserve began to dwindle.
The U.S. dollar has in the past seven years reduced its weight in the basket of currencies by 20%. It has been observed that central banks of countries are protecting their reserves from the falling dollar by buying the euro and the yen. The falling dollar also introduced questions as to whether the US deliberately devalued the dollar so as to reduce its very high public debt, whose creditors are some of the largest economies in the world - China and Japan.

It is quite likely that in future the demand for the euro and the yen will continue to grow and the share of dollars in reserves will fall. Obviously such a process will continue gradually, as a sharp decline in USD may affect the stability of world monetary system. At the same time, one observes a trend of diversification of foreign exchange and gold reserves and to achieve this goal one has to reduce holdings of dollars. But experts doubt the possibility of replacing national currencies or the dollar or euro by SDRs. It really is not a new currency. It has no advantage to the functioning of national and supranational currencies such as Euros. It seems that the SDR will not become world money mainly because it does not have an infrastructure and because the dollar is firmly rooted in the global economy. In order to replace the national currencies in international monetary functions, one would have to emit SRDs on an incredible scale. In addition, changes in the financial system of the world are possible, but require a consensus between the major economies.

In parallel with the ideas of replacing the dollar with SDRs there appeared other ideas such as creating a single currency or even for groups of countries, such as the Arab states, Russia and China. To be economical, these proposals must fulfill the requirements of optimum currency areas. None of these countries meet such a condition of monetary union, and getting rid of their currency would create a crisis similar to that in Argentina. An original proposal was presented in June at the International Economic Forum in St. Petersburg, by Russian Finance Minister Alexei Kudrin. He proposed that the Chinese Yuan could become the world's reserve currency after achieving full convertibility, which will take about 10 years. Is it a realistic idea? - It is unlikely that the U.S. dollar will be replaced by either SDRs or by any other rival currency. However, there might be a change towards a greater diversification of foreign exchange reserves. There may appear a new multi-currency system. However, the question arises whether such a system based on a basket of different currencies with varying prices will not be destabilizing to financial markets. The problem will hinge on maintaining the stability of the system in the event of a loss of dominance of the dollar. The role of international and national regulation of international financial institutions will likely expand.

In January of 2010 opinions appeared that interest in the dollar is back. One might venture the opinion that the financial world paid too much attention to the fate of the U.S. currency while turning a blind eye to the fact that in Europe the situation is similar: the governments of other countries led a stimulating fiscal policy introducing huge amounts of money into the financial system. As a result, this always leads to an increase in spending. Financial problems have occurred in Iceland, Latvia, Ireland, Portugal, Greece and Spain. These events and the danger of the emergence of similar scenarios in other countries of the euro area became a turning moment. Anti-dollar moods gave way to worries about the Euro.

---

1 The greatest geographical area in which economic activity would be maximised if a common currency were introduced. The author of this concept is the Canadian economist Robert Mundell, Nobel Laureate in Economics 1999.
As a result, confidence in the dollar began to rise. On the need for a strong dollar talked the President of the European Central Bank Jean-Claude Trichet. Such an opinion may betray a view of a too expensive euro which adversely affects the export-oriented economy of the EU. It should be asked whether the belief in the rebirth of the global economy is not too eager. Investors today think about the condition of the global economy, which, after the cutoff in the inflow of cheap money, may once again surprise all the optimists.

3. A NEW CONCEPT FOR CURRENCY BASKET OPTIMIZATION

3.1. Problem formulation

Let $P_{ki}$ denote the time series of prices (in USD/mass unit) of a primary commodity $k$, at time instants $i=n-N+1$, $n-N+2$, ..., $n$, where $N$ is the series length taken arbitrarily (e.g. daily close quotations at a stock exchange in the time interval containing $N$ working days), $R_{ci}$ – the series of an official exchange rate quotations of $c$-th currency (in currency unit/USD) recorded at the same time instants as $P_{ki}$ [11].

Let us consider a trade contract made at time $n$, concerning a commodity $k$, to be delivered at time $n+p$. One can take the agreement (Rule I) the contract amount due may be paid at time $n$ or $n+p$ with a package of quota $V_k=\{V_{kc}, c=1, ..., C\}$ in different currencies

$$V_{ken} = P_{kn} \beta_c R_{cn} \quad \sum_{c=1}^{C} \beta_c = 1 \quad \sum_{c=1}^{C} \frac{V_{ken}}{R_{cn}} \equiv P_{kn} \quad (1)$$

where $\beta_c$ means the fraction of the original price to be paid in $c$-th currency, agreed at time $n$ or before. The quota $V_k$ are fixed at the time $n$ according to eq.(1), so that $V_{ken+p} = V_{ken}$.

The transaction risk [12] could be expressed as the change $\Delta_1 P_{kp}$ of the commodity price recalculated to US dollars at the time $n+p$:

$$\Delta_1 P_{kp} = P_{kvp} - \sum_{c=1}^{C} \frac{V_{ken}}{R_{cn+p}} - \sum_{c=1}^{C} \beta_c \frac{R_{cn}}{R_{cn+p}} P_{kn} \quad (2)$$

One can take also another rule (Rule II): at the time $n$ we define only a currency basket $W_n=\{W_{cn}; c=1, ..., C\}$ where $W_{cn} = b_c R_{cn}$ is the quota of $c$-th currency to be paid for 1 USD, either at time $n$ or $n+p$. The transaction risk may be expressed as the difference $\Delta_II P_{kp}$ of the commodity price paid at $n$ and $n+p$, recalculated to US dollars at the time $n+p$:

$$\Delta_II P_{kp} = (P_{kvp} - P_{kn}) \sum_{c=1}^{C} \beta_c \frac{R_{cn}}{R_{cn+p}} \quad (3)$$

Nevertheless, the risk measures (2) and (3) may be misleading, as they do not take into account changeable position (appreciation/depreciation) of the USD itself. Moreover the risk assessment involves the ratio of two random variables $R_{cn}/R_{cn+p}$ that makes it more uncertain.
Hence, to eliminate the above drawbacks we propose to use for the trade risk assessment an instrumental price $\Pi_{kn}$, based on the currency basket composed of the currencies $c=1, \ldots, C$, recalculated to USD with constant exchange ratios $R_{cref}$.

$$\Pi_{kn} = P_{kn} \sum_{c=1}^{C} b_c \frac{R_c}{R_{cref}} \quad \sum_{c=1}^{C} b_c = 1$$  \hspace{1cm} (4)

where \{b_c: c=1, \ldots, C\} are the factors (the basket coefficients) partitioning the transaction risk onto the currencies $c$.

The contract can be made according to the rules I or II with $\beta_c = b_c$, but its risk may be evaluated as the instrumental price change:

$$\Delta_1 \Pi_{kp} = \sum_{c=1}^{C} \left( b_c \frac{P_{kn}}{R_{cref}} \right) \left( P_{kn+p} R_{cref} - P_{kn} R_{cref} \right)$$  \hspace{1cm} (5)

or weighted change of the original price (like in eq.(3):

$$\Delta_II \Pi_{kp} = (P_{kn+p} - P_{kn}) \sum_{c=1}^{C} b_c R_{cref}$$  \hspace{1cm} (6)

The above measures express better the contract risk than eqs (2,3), particularly when the quota $V_{kn}$ to be paid at time $n$ had been acquired in a longer time interval (not bought at time $n$), which is rather typical case. Hence, the most suitable reference exchange rate $R_{cref}$ seems to be the mean value $R_{cref}$ in a presumed time interval containing $N$ historical samples of $R_{ci}$ and ending at $L$-th sample ($i=LN+1, \ldots, L$), with $L$ taken arbitrarily (NL interval)

$$R_{cref} = R_{cref}^{def} = \frac{1}{N} \sum_{i=1}^{N} R_{cref_{i+1}}$$  \hspace{1cm} (7)

The currency basket coefficients $b_c$ may be adjusted in such a way, to minimize the overall trade risk, expressed by the variance of $\Delta_1 \Pi_{kp}$ or $\Delta_II \Pi_{kp}$ in NL interval, averaged over the set of the commodities to be sale/buy with the same basket. To this aim the linear quadratic optimization tools may be applied, minimizing one of the above performance measures:

$$J_{NL} = \frac{1}{K} \sum_{k=1}^{K} \left( \frac{1}{N} \sum_{i=0}^{N-1} \sum_{c=1}^{C} \left( b_c \frac{P_{i+1} R_{L+i} - P_{i} R_{L+i}}{R_{cref_{i+1}}} \right)^2 \right)$$  \hspace{1cm} (8)

$$J_{WNL} = \frac{1}{K} \sum_{k=1}^{K} \left( \frac{1}{N} \sum_{i=0}^{N-1} \sum_{c=1}^{C} \left( \frac{b_c}{R_{cref_{i+1}}} R_{L+i} (P_{i+1} - P_{i}) \right)^2 \right)$$  \hspace{1cm} (9)
If the delivery delay $p$ is differentiated or varying, the adequate averaged measure of the trade risk is simply the standard deviation (variance) of $\Pi_{i\ell}$ in the NL interval. Thus, the third alternative performance index may be calculated as follows:

$$J_{\text{NL}} = \frac{1}{K} \sum_{k=0}^{K} \left( \frac{1}{N} \sum_{n=0}^{N-1} \sum_{c=1}^{C} b_c R_{NL}^c \right)^2 - \left( \frac{1}{C} \sum_{c=1}^{C} \left( \frac{1}{N} \sum_{n=0}^{N-1} R_{NL}^c \right) \right)^2$$

(10)

The following constraints must be satisfied:

$$b_c \geq 0 \quad \text{for} \quad c=1, \ldots, C, \quad \text{and} \quad \sum_{c=1}^{C} b_c = 1$$

(11)

The basket may be constructed individually for each transaction by solving the above optimization task, defined for a given NL interval (e.g. $L=n$, and $N$ covering a couple of months or years). When using typical, commonly available software tools (we have employed \texttt{fmincon} function working under MATLAB software package) it takes a few seconds. However, to make the trade principles stable and predictable, the basket should be optimized over properly large time interval ($N$ covering two or more years), in which the random fluctuations in the market can be well averaged (notice that the SDR basket is calculated for the period of 5 years at present), and then applied in properly long time $n=L+p$, ..., $L+p+M$ ($M$ covers at least one year), or $n=L$, ..., $L+M$, if the performance index (10) was employed.

Current transactions risk may be assessed by calculation the instrumental price variance (or standard deviation) with eqs (8-10) in a representative interval, ending at current time $n$ or $n-p$ and containing properly large number of historical samples (100 or more).

3.2. Data characterization

In the study we have used the daily close prices of six raw materials: Aluminium, Copper, Lead, Nickel, Tin, Zinc and daily interbank exchange rates of nine currencies: Australian Dollar, Brazilian Real, British Pound, Canadian Dollar, Euro, Indian Rupee, Japanese Yen, Polish Zloty, Russian Ruble, U.S. Dollar. The data were recorded in the time interval from 01.01.1998 to 2.02.2010. Both exchange rates and raw material prices were stated in US dollar (currency per USD, USD per given raw material unit of measure) [13].

Regarding the goal of the analyses being carried out to construct an alternative basket towards the current SDR basket, there was also SDR exchange rate (founded on the mentioned basket) expressed by USD (SDR/USD) in the gathered data. Moreover, we have taken into account possible using of Gold and Silver as interesting alternatives for other currencies (with their exchange rates expressed in mass unit per USD). The currency set extended in such a way gives more flexibility in risky financial operations. It also gives a possibility to check in what degree the current SDR basket is fitted to the current situation in the world economy.

Time series of the examined raw material prices and exchange rates are described in Table 1.
### Raw Materials Prices

<table>
<thead>
<tr>
<th>Raw Materials Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alumin</td>
</tr>
<tr>
<td>Aluminum, London Metal Exchange, daily official prices mean cash, Dollars per Ton</td>
</tr>
<tr>
<td>Copper</td>
</tr>
<tr>
<td>Copper, London Metal Exchange, daily settlement prices, Dollars per Ton</td>
</tr>
<tr>
<td>Lead</td>
</tr>
<tr>
<td>Lead, London Metal Exchange, daily official prices mean cash, Dollars per Ton</td>
</tr>
<tr>
<td>Nickel</td>
</tr>
<tr>
<td>Nickel, London Metal Exchange, daily official prices mean cash, Dollars per Ton</td>
</tr>
<tr>
<td>Tin</td>
</tr>
<tr>
<td>Tin, London Metal Exchange, daily official prices mean cash, Dollars per Ton</td>
</tr>
<tr>
<td>Zinc</td>
</tr>
<tr>
<td>Zinc, London Metal Exchange, daily official prices mean cash, Dollars per Ton</td>
</tr>
<tr>
<td>Silver</td>
</tr>
<tr>
<td>London Bullion Market Association, held each working day at 12:00 PM in the City of London, Dollars per Troy Ounce</td>
</tr>
<tr>
<td>Gold</td>
</tr>
<tr>
<td>London Bullion Market Association, Gold prices Day 3:00 PM, Dollars per Troy Ounce</td>
</tr>
</tbody>
</table>

### Exchange rates

<table>
<thead>
<tr>
<th>Exchange rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD/USD</td>
</tr>
<tr>
<td>Australian Dollar/U.S. Dollar</td>
</tr>
<tr>
<td>BRL/USD</td>
</tr>
<tr>
<td>Brazilian Real/ U.S. Dollar</td>
</tr>
<tr>
<td>GBP/USD</td>
</tr>
<tr>
<td>British Pound/U.S. Dollar</td>
</tr>
<tr>
<td>CAD/USD</td>
</tr>
<tr>
<td>Canadian Dollar/U.S. Dollar</td>
</tr>
<tr>
<td>EUR/USD</td>
</tr>
<tr>
<td>Euro/U.S. Dollar</td>
</tr>
<tr>
<td>INR/USD</td>
</tr>
<tr>
<td>Indian Rupee/U.S. Dollar</td>
</tr>
<tr>
<td>JPY/USD</td>
</tr>
<tr>
<td>Japanese Yen/U.S. Dollar</td>
</tr>
<tr>
<td>PLN/USD</td>
</tr>
<tr>
<td>Polish Zloty/U.S. Dollar</td>
</tr>
<tr>
<td>RUB/USD</td>
</tr>
<tr>
<td>Russian Rouble/US Dollar</td>
</tr>
<tr>
<td>SDR/USD</td>
</tr>
<tr>
<td>Special Drawing Right/ U.S. Dollar</td>
</tr>
</tbody>
</table>

Numerical treatment of the data with software tools used in our research faces two technical problems. The first one is incoherency of data registration period (companies come into the stock exchange and then sometimes leave it suddenly), the second one comes from deficiency of data (for example weekends, holidays). Weekends are synchronical interruptions and that’s the reason why we can erase them and regard as continuous period. Asynchronical deficiencies (holidays or global incidents such as terrorist attack on WTC or U.S. intervention in Iraq) cause mainly interruption which lasts couple days or more and effects the work of stock exchange. This kind of deficiencies can’t be eliminated in the same way as synchronical. Nevertheless we can eliminate them in two different ways. The classic one which rests upon interpolation of shortages and helps to gain compact set of data. The second one is based on ignoring data insufficiencies [1, 14]. The problem concerning incoherency of the data was solved in through erasing weekend days, while deficiencies resulted from different causes were removed through linear interpolation.

The time profiles of studied series are shown in figures 1 and 2. As it is presented all studied series are nonstationary. Qualitatively similar shape of the time profiles of the non-iron metals prices quotation (Fig. 2) is characteristic what perhaps could show their correlation.
Fig. 1. Time series of non-iron metals prices (London Metal Exchange) used in calculations. The values in each series are related to their maximal value. Vertical dotted lines – three months and 1-year (bold) intervals.

Fig. 2. Time series of exchange rates of currencies used in calculations. The values in each series are related to their maximal value. Vertical dotted lines – three months and 1-year (bold) intervals.
3.3. Computation results

The calculations were performed with our own software working under MATLAB software platform, employing MATLAB \texttt{fmincon()} function as the solver of the optimization task (8-11).

The currency basket has been optimized in four-years intervals (1044 samples). The interval length corresponds to cyclic properties of World economy [14]. Significant contribution of four-year cycles to financial time series is often suggested in literature (see [15, 16]). In our earlier papers [1, 2, 14] we have shown the eight-year cycles in leading Stock Market indices and metal prices are also present, but four-years oscillations are of significance too. Thus the interval covering four years data seems to be a good compromise between filtering (averaging) and flexibility properties of the numerical analysis. The optimised basket has been employed in one-year interval, then recalculated in the interval shifted ahead by this year.

The currency basket has been optimized for delivery delay \(p=0\) (minimization of the instrumental price variance – eq.(10), then for \(p=22\) (1 month), \(p=261\) (1 year) and \(p=522\) (2 years) of present dollar currency like in the formula (2). Also in case of the \(W_{on}\) currency it was decided that the basket \(K\) will be calculated for one year interval in four years window.

Hereinafter presented results were gained with using a dedicated package of software, working in the MATLAB environment. Four different sets of data containing the exchange rates were used in the study. Table 2 presents these sets.

\textit{Table 2.}

<table>
<thead>
<tr>
<th>Variant No</th>
<th>Currency set</th>
<th>Delivery delay (p)</th>
<th>Performance index</th>
<th>Presentation of results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>full</td>
<td>(p=0)</td>
<td>(\Pi) variance – eq.(10)</td>
<td>Figs 4, 7, Table 4.</td>
</tr>
<tr>
<td>2</td>
<td>full</td>
<td>(p=22) (1 m)</td>
<td>(\Delta \Pi) variance – eq.(8)</td>
<td>Figs 3, 6, Table 3.</td>
</tr>
<tr>
<td>3</td>
<td>full</td>
<td>(p=22) (1 m)</td>
<td>(\Delta \Pi) variance – eq.(9)</td>
<td>Figs 5, 8, Table 5.</td>
</tr>
<tr>
<td>4</td>
<td>full</td>
<td>(p=261) (1 y)</td>
<td>(\Delta \Pi) variance – eq.(8)</td>
<td>Fig. 9, Table 6.</td>
</tr>
<tr>
<td>5</td>
<td>full</td>
<td>(p=261) (1 y)</td>
<td>(\Delta \Pi) variance – eq.(9)</td>
<td>Fig. 10, Table 7.</td>
</tr>
<tr>
<td>6</td>
<td>full</td>
<td>(p=522) (2 y)</td>
<td>(\Delta \Pi) variance – eq.(8)</td>
<td>Fig. 11, Table 8.</td>
</tr>
<tr>
<td>7</td>
<td>full</td>
<td>(p=522) (2 y)</td>
<td>(\Delta \Pi) variance – eq.(9)</td>
<td>Fig. 12, Table 9.</td>
</tr>
<tr>
<td>8</td>
<td>Silver and Gold excluded</td>
<td>(p=0)</td>
<td>(\Pi) variance – eq.(10)</td>
<td>Fig. 13, Table 11.</td>
</tr>
<tr>
<td>9</td>
<td>Silver and Gold excluded</td>
<td>(p=22) (1 m)</td>
<td>(\Delta \Pi) variance – eq.(8)</td>
<td>Table 10.</td>
</tr>
<tr>
<td>10</td>
<td>Silver and Gold excluded</td>
<td>(p=22) (1 m)</td>
<td>(\Delta \Pi) variance – eq.(9)</td>
<td>Table 12.</td>
</tr>
<tr>
<td>11</td>
<td>Silver and Gold excluded</td>
<td>(p=522) (2 y)</td>
<td>(\Delta \Pi) variance – eq.(8)</td>
<td>Table 13.</td>
</tr>
<tr>
<td>12</td>
<td>Silver and Gold excluded</td>
<td>(p=522) (2 y)</td>
<td>(\Delta \Pi) variance – eq.(9)</td>
<td>Table 14.</td>
</tr>
</tbody>
</table>
To get an insight into a mechanism of the risk reduction we calculated time profiles $\Omega_n$ of the instrumental currency (unit/USD) based on the optimized baskets:

$$\Omega_n = \sum b_c R_{c} / R_{LN} + \Delta_\Omega$$

(12)

where $\Delta_\Omega$ is a shift added to keep continuity of the currency value after the basket reconstruction (it has no importance in practice, as in contracts the value for $\Omega$ is always calculated with a given basket defined before). Initial value $\Omega_0$ was fixed at value of SDR$_0$. Time profiles of $\Omega_n$, Gold/USD and SDR/USD are compared in Figs 3-6. It may be seen the $\Omega_n$ is similar to Gold. Both, $\Omega_n$ and Gold are of decreasing tendencies, although $\Omega_n$ is more varying, and so, better fitted to changes in commodities prices. SDR profile is different (rather growing).

![Fig.3. Time profiles of the basket currency Cur.opt, SDR, Gold exchange rates. The basket optimised for p=1 month, with performance index defined in eq.(8)](image)

![Fig.4. Time profiles of the basket currency Cur.opt, SDR, Gold exchange rates. The basket optimised for p=0, with performance index defined in eq.(10)](image)

![Fig.5. Time profiles of the basket currency Cur.opt, SDR, Gold exchange rates. The basket optimised for p=1 month, with performance index defined in eq.(9)](image)

Pictures shown in Figures 6-13 compare time profiles of the prices calculated for the commodities involved in the basket optimization: the optimal instrumental price $\Pi$ (dark lines) calculated out of NL intervals (with baskets found before), in USD (grey lines), in SDR (point grey lines), in Gold unit (dotted grey lines). One can see that $\Pi$ is in general less varying that other prices, but it is similar to that in Gold. Nevertheless it is noticeably better than the prices in USD and SDR.
Fig. 6. Time profiles of non-iron metals instrumental prices (OPT) calculated with the basket minimizing the function (8) with $p=22$ (one month), compared with original prices (USD) and prices in Gold mass units (Gold), all related to their maximal value.

Fig. 7. Time profiles of non-iron metals instrumental prices (OPT) calculated with the basket minimizing the function (10) $p=0$, compared with original prices (USD) and prices in Gold mass units (Gold), all related to their maximal value.
Fig. 8. Time profiles of non-iron metals instrumental prices (OPT) calculated with the basket minimizing the function (9) $p=22$, compared with original prices (USD) and prices in Gold mass units (Gold), all related to their maximal value.

Fig. 9. Time profiles of non-iron metals instrumental prices (OPT) calculated with the basket minimizing the function (8) $p=261$ (1 year), compared with original prices (USD) and prices in Gold mass units (Gold), all related to their maximal value.
Fig. 10. Time profiles of non-iron metals instrumental prices (OPT) calculated with the basket minimizing the function (9) $p=261$ (1 year) with Silver and Gold excluded, compared with original prices (USD) and prices in Gold mass units (Gold), all related to their maximal value.

Fig. 11. Time profiles of non-iron metals instrumental prices (OPT) calculated with the basket minimizing the function (8) $p=261$ (1 year) with Silver and Gold excluded, compared with original prices (USD) and prices in Gold mass units (Gold), all related to their maximal value.
**Fig. 12.** Time profiles of non-iron metals instrumental prices (OPT) calculated with the basket minimizing the function (8) $p=522$ (2 years), compared with original prices (USD) and prices in Gold mass units (Gold), all related to their maximal value.

**Fig. 13.** Non-iron metals instrumental prices (Bask. OPT) calculated with the basket minimizing the function (9) $p=521$ (2 years), compared with original prices (USD) and prices in Gold mass units (Gold), all related to their maximal value.
Differences in the risk linked with the above prices are characterized quantitatively in Tables 3–14. The optimal basket structures obtained in different variants of calculations (see Table 2) are also presented.

The first row in Tables below shows the first day, from which the optimal basket was available (starting day of the basket application interval). The consecutive pairs of rows (2,3; 4,5; …; 10,11) compare (in consecutive time intervals) the transaction risk measures, averaged over the set of the commodities considered, calculated for prices in more important currencies (USD, EURO, SDR, Silver, Gold) and related to that reached with the optimal basket (the risk ratio is denoted as USD/Bask, …, Gold/Bask). The upper row in each pair shows the values obtained in the basket optimization interval (4 years), the lower row – the values calculated in the basket application interval (1 year). In tables 10-14 only the lower row is given, as the Gold and Silver were excluded from the basket (values for optimization intervals are unavailable).

In the optimization intervals (upper rows) the risk of the basket transactions (Bask) is the square root of the performance index used in the variant presented (p=0 - eq.(10), p>0 – eq.(8) or eq.(9)). Correspondingly, the risk of the homogeneous prices (in USD, EURO, …) is calculated as the standard deviation of the price (if p=0) or p−sample price returns (if p>0), related to mean value of the currency exchange rate (i.e. $\Pi$ or $\Delta \Pi$ with homogeneous baskets). Thus, the numbers in the upper row show how far the optimal basket is better (in sense of the applied performance index) than the homogeneous one (obviously, the numbers are greater than 1, as the optimal basket gives always lower values).

For application intervals (numbers in the lower row) we have use a more intuitive measure of transaction risk, namely, the standard deviation of the price expressed in the currency considered, related to its mean value (it is insensitive to differences in the level of particular prices). Notice that for the optimal basket (Bask) such a risk measure may be greater than for other currencies (the values may be lower than 1), as: 1) it refers to the price variance (which is minimized only in the variants with p=0); 2) even if the variants with p=0 are considered, the optimal basket (found in the optimization interval) may not be optimal in the application interval (due to changes in the series properties). Thus, the risk ratio greater than 1 appearing in the lower row means that the optimized basket is certainly advantageous when compared to the homogeneous currencies.

Further rows in Tables 3-14 present (in columns) the optimal basket structures found in the consecutive time intervals. In Tables 4-14 the currencies not contributing into the basket in all the intervals are omitted (to make the presentation more clear).

Optimal basket risk properties: delay $p=1m$, performance index – eq.(8), full currency set.

<table>
<thead>
<tr>
<th>Last data</th>
<th>6.02.02</th>
<th>6.02.03</th>
<th>6.02.04</th>
<th>6.02.05</th>
<th>6.02.06</th>
<th>6.02.07</th>
<th>6.02.08</th>
<th>6.02.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD/Bask</td>
<td>1.037</td>
<td>1.041</td>
<td>1.193</td>
<td>1.264</td>
<td>1.192</td>
<td>1.351</td>
<td>1.297</td>
<td>1.207</td>
</tr>
<tr>
<td></td>
<td>0.955</td>
<td>1.404</td>
<td>1.170</td>
<td>1.155</td>
<td>1.136</td>
<td>1.021</td>
<td>1.222</td>
<td>1.620</td>
</tr>
<tr>
<td>EUR/Bask</td>
<td>1.247</td>
<td>1.171</td>
<td>1.125</td>
<td>1.083</td>
<td>1.051</td>
<td>1.262</td>
<td>1.223</td>
<td>1.136</td>
</tr>
<tr>
<td></td>
<td>1.306</td>
<td>0.974</td>
<td>0.995</td>
<td>1.247</td>
<td>1.020</td>
<td>1.016</td>
<td>0.983</td>
<td>1.281</td>
</tr>
<tr>
<td>SDR/Bask</td>
<td>1.008</td>
<td>1.046</td>
<td>1.329</td>
<td>1.410</td>
<td>1.306</td>
<td>1.418</td>
<td>1.337</td>
<td>1.255</td>
</tr>
<tr>
<td></td>
<td>1.213</td>
<td>1.709</td>
<td>1.349</td>
<td>1.121</td>
<td>1.204</td>
<td>1.043</td>
<td>1.343</td>
<td>1.817</td>
</tr>
</tbody>
</table>
### Table 4.
Optimal basket risk properties: delay \( p = 0 \), performance index – eq. (10), full currency set.

<table>
<thead>
<tr>
<th>Last data</th>
<th>6.02.02</th>
<th>6.02.03</th>
<th>6.02.04</th>
<th>6.02.05</th>
<th>6.02.06</th>
<th>6.02.07</th>
<th>6.02.08</th>
<th>6.02.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD/Bask</td>
<td>1.204</td>
<td>1.275</td>
<td>1.558</td>
<td>1.850</td>
<td>1.823</td>
<td>2.451</td>
<td>2.118</td>
<td>1.494</td>
</tr>
<tr>
<td>EUR/Bask</td>
<td>1.452</td>
<td>1.370</td>
<td>1.201</td>
<td>1.161</td>
<td>1.281</td>
<td>2.209</td>
<td>1.928</td>
<td>1.394</td>
</tr>
<tr>
<td>SDR/Bask</td>
<td>1.223</td>
<td>1.361</td>
<td>1.856</td>
<td>2.226</td>
<td>2.105</td>
<td>2.556</td>
<td>2.192</td>
<td>1.548</td>
</tr>
<tr>
<td>Silver/Bask</td>
<td>1.252</td>
<td>1.124</td>
<td>1.118</td>
<td>1.002</td>
<td>1.002</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Gold/Bask</td>
<td>1.233</td>
<td>1.367</td>
<td>1.280</td>
<td>1.082</td>
<td>1.212</td>
<td>1.379</td>
<td>1.278</td>
<td>1.245</td>
</tr>
</tbody>
</table>

| Basket |
|--------|---------|---------|---------|---------|---------|---------|---------|---------|
| JPY/USD| 0.966   | 0.924   | 0.000   | 0.000   | 0.000   | 0.000   | 0.000   | 0.000   |
| BRL/USD| 0.034   | 0.076   | 0.000   | 0.000   | 0.000   | 0.000   | 0.000   | 0.000   |
| AUD/USD| 0.000   | 0.000   | 1.000   | 0.000   | 0.097   | 0.000   | 0.000   | 0.000   |
| Silver/USD | 0.000 | 0.000   | 0.000   | 0.000   | 0.872   | 0.903   | 1.000   | 1.000   |
| Gold/USD | 0.000   | 0.000   | 0.000   | 0.128   | 0.000   | 0.000   | 0.000   | 0.000   |

### Table 5.
Optimal basket risk properties: delay \( p = -22 \), performance index – eq. (9), full currency set.

<table>
<thead>
<tr>
<th>Last data</th>
<th>6.02.02</th>
<th>6.02.03</th>
<th>6.02.04</th>
<th>6.02.05</th>
<th>6.02.06</th>
<th>6.02.07</th>
<th>6.02.08</th>
<th>6.02.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD/Bask</td>
<td>1.029</td>
<td>1.023</td>
<td>1.105</td>
<td>1.174</td>
<td>1.118</td>
<td>1.289</td>
<td>1.272</td>
<td>1.136</td>
</tr>
<tr>
<td>EUR/Bask</td>
<td>1.070</td>
<td>1.030</td>
<td>1.017</td>
<td>1.038</td>
<td>1.046</td>
<td>1.249</td>
<td>1.228</td>
<td>1.114</td>
</tr>
</tbody>
</table>

| Basket |
|--------|---------|---------|---------|---------|---------|---------|---------|---------|
| JPY/USD| 1.107   | 1.011   | 0.950   | 1.641   | 1.139   | 1.012   | 1.218   | 1.174   |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| SDR/Bask  | 1.023     | 1.027     | 1.163     | 1.248     | 1.153     | 1.305     | 1.287     | 1.144     |           |           |           |           |           |
|           | 1.027     | 1.773     | 1.288     | 1.476     | 1.345     | 1.039     | 1.664     | 1.665     |           |           |           |           |           |
| Silver/Bask | 1.043     | 1.013     | 1.012     | 1.000     | 1.000     | 1.000     | 1.000     |           |           |           |           |           |           |
|           | 0.870     | 0.765     | 1.415     | 1.083     | 1.136     | 1.208     | 1.011     | 1.003     |           |           |           |           |           |
| Gold/Bask | 1.028     | 1.020     | 1.006     | 1.020     | 1.031     | 1.079     | 1.067     | 1.037     |           |           |           |           |           |
|           | 1.014     | 0.928     | 1.004     | 1.109     | 1.211     | 1.331     | 1.453     | 1.140     |           |           |           |           |           |

| JPY/USD   | 1.000     | 0.843     | 0.000     | 0.000     | 0.000     | 0.000     | 0.000     | 0.000     |           |           |           |           |           |
| BRL/USD   | 0.000     | 0.157     | 0.000     | 0.000     | 0.000     | 0.000     | 0.000     | 0.000     |           |           |           |           |           |
| AUD/USD   | 0.000     | 0.000     | 1.000     | 0.000     | 0.000     | 0.000     | 0.000     | 0.000     |           |           |           |           |           |
| Silver/USD | 0.000     | 0.000     | 0.000     | 1.000     | 1.000     | 1.000     | 1.000     | 1.000     |           |           |           |           |           |

Table 6.

Optimal basket risk properties: delay \( p = 261 \), performance index – eq.(8), full currency set.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SDR/Bask</td>
<td>1.021</td>
<td>1.145</td>
<td>1.677</td>
<td>1.483</td>
<td>2.609</td>
<td>1.920</td>
<td>1.355</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.425</td>
<td>1.163</td>
<td>1.318</td>
<td>1.264</td>
<td>1.025</td>
<td>1.416</td>
<td>1.345</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EUR/Bask</td>
<td>1.049</td>
<td>1.012</td>
<td>1.228</td>
<td>1.226</td>
<td>2.385</td>
<td>1.764</td>
<td>1.267</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.015</td>
<td>1.030</td>
<td>1.497</td>
<td>1.119</td>
<td>1.006</td>
<td>1.156</td>
<td>1.054</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SDR/Bask</td>
<td>1.740</td>
<td>1.314</td>
<td>1.267</td>
<td>1.349</td>
<td>1.053</td>
<td>1.547</td>
<td>1.508</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.048</td>
<td>1.081</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silver/Bask</td>
<td>0.843</td>
<td>1.435</td>
<td>1.095</td>
<td>1.114</td>
<td>1.173</td>
<td>0.925</td>
<td>0.893</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gold/Bask</td>
<td>1.000</td>
<td>1.000</td>
<td>1.159</td>
<td>1.170</td>
<td>1.611</td>
<td>1.290</td>
<td>1.052</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.895</td>
<td>1.078</td>
<td>1.099</td>
<td>1.191</td>
<td>1.257</td>
<td>1.306</td>
<td>1.048</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Basket    |            |            |            |            |            |            |            |            |           |           |           |           |           |
| JPY/USD   | 1.000     | 0.000     | 0.000     | 0.000     | 0.000     | 0.000     | 0.000     | 0.000     |           |           |           |           |           |
| AUD/USD   | 0.000     | 1.000     | 0.191     | 0.289     | 0.000     | 0.000     | 0.000     | 0.000     |           |           |           |           |           |
| Silver/USD | 0.000     | 0.000     | 0.809     | 0.711     | 1.000     | 1.000     | 1.000     |           |           |           |           |           |           |

Table 7.

Optimal basket risk properties: delay \( p = -261 \), performance index – eq.(9), full currency set.
### Table 8.

<table>
<thead>
<tr>
<th>Last data</th>
<th>6.02.04</th>
<th>6.02.05</th>
<th>6.02.06</th>
<th>6.02.07</th>
<th>6.02.08</th>
<th>6.02.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD/Bask</td>
<td>1.692</td>
<td>2.123</td>
<td>2.583</td>
<td>3.364</td>
<td>8.425</td>
<td>2.788</td>
</tr>
<tr>
<td>EUR/Bask</td>
<td>1.053</td>
<td>1.107</td>
<td>1.199</td>
<td>1.022</td>
<td>1.274</td>
<td>1.215</td>
</tr>
<tr>
<td>SDR/Bask</td>
<td>1.178</td>
<td>1.121</td>
<td>1.606</td>
<td>3.003</td>
<td>7.465</td>
<td>2.417</td>
</tr>
<tr>
<td>Silver/Bask</td>
<td>0.935</td>
<td>1.261</td>
<td>1.061</td>
<td>1.002</td>
<td>1.041</td>
<td>0.952</td>
</tr>
<tr>
<td>Gold/Bask</td>
<td>2.051</td>
<td>2.707</td>
<td>3.131</td>
<td>3.516</td>
<td>8.769</td>
<td>2.978</td>
</tr>
<tr>
<td></td>
<td>1.187</td>
<td>1.063</td>
<td>1.279</td>
<td>1.050</td>
<td>1.392</td>
<td>1.362</td>
</tr>
<tr>
<td></td>
<td>1.208</td>
<td>1.017</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>1.296</td>
<td>0.926</td>
<td>1.053</td>
<td>1.167</td>
<td>0.834</td>
<td>0.805</td>
</tr>
<tr>
<td></td>
<td>1.251</td>
<td>1.055</td>
<td>1.485</td>
<td>1.608</td>
<td>3.183</td>
<td>1.614</td>
</tr>
<tr>
<td></td>
<td>0.980</td>
<td>0.928</td>
<td>1.127</td>
<td>1.248</td>
<td>1.175</td>
<td>0.945</td>
</tr>
<tr>
<td>Basket</td>
<td>1.000</td>
<td>0.482</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Silver/USD</td>
<td>1.000</td>
<td>0.518</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

### Table 9.

<table>
<thead>
<tr>
<th>Last date</th>
<th>6.02.04</th>
<th>6.02.05</th>
<th>6.02.06</th>
<th>6.02.07</th>
<th>6.02.08</th>
<th>6.02.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD/Bask</td>
<td>1.350</td>
<td>2.039</td>
<td>3.268</td>
<td>8.679</td>
<td>0.000</td>
<td>1.842</td>
</tr>
<tr>
<td>EUR/Bask</td>
<td>1.135</td>
<td>1.258</td>
<td>1.218</td>
<td>1.023</td>
<td>1.308</td>
<td>1.245</td>
</tr>
<tr>
<td>SDR/Bask</td>
<td>1.098</td>
<td>1.131</td>
<td>2.130</td>
<td>7.840</td>
<td>0.000</td>
<td>1.626</td>
</tr>
<tr>
<td>Silver/Bask</td>
<td>1.008</td>
<td>1.433</td>
<td>1.078</td>
<td>1.004</td>
<td>1.069</td>
<td>0.976</td>
</tr>
<tr>
<td>Gold/Bask</td>
<td>1.474</td>
<td>2.497</td>
<td>3.830</td>
<td>9.026</td>
<td>0.000</td>
<td>1.940</td>
</tr>
<tr>
<td></td>
<td>1.280</td>
<td>1.209</td>
<td>1.299</td>
<td>1.051</td>
<td>1.428</td>
<td>1.395</td>
</tr>
<tr>
<td></td>
<td>1.206</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>1.398</td>
<td>1.052</td>
<td>1.070</td>
<td>1.169</td>
<td>0.856</td>
<td>0.825</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>1.035</td>
<td>1.962</td>
<td>4.275</td>
<td>0.000</td>
<td>1.091</td>
</tr>
<tr>
<td></td>
<td>1.057</td>
<td>1.055</td>
<td>1.145</td>
<td>1.250</td>
<td>1.206</td>
<td>0.968</td>
</tr>
<tr>
<td>Basket</td>
<td>0.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Silver/USD</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Gold/USD</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

### Table 10.
Optimal basket risk properties: delay $p=22$, Silver and Gold excluded – eq.(8),

<table>
<thead>
<tr>
<th>Last date</th>
<th>6.02.02</th>
<th>6.02.03</th>
<th>6.02.04</th>
<th>6.02.05</th>
<th>6.02.06</th>
<th>6.02.07</th>
<th>6.02.08</th>
<th>6.02.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD/Bask</td>
<td>1.033</td>
<td>1.034</td>
<td>1.156</td>
<td>1.228</td>
<td>1.159</td>
<td>1.124</td>
<td>1.134</td>
<td>1.109</td>
</tr>
<tr>
<td></td>
<td>0.912</td>
<td>1.236</td>
<td>1.123</td>
<td>0.963</td>
<td>1.080</td>
<td>1.007</td>
<td>1.366</td>
<td>1.543</td>
</tr>
<tr>
<td>EUR/Bask</td>
<td>1.243</td>
<td>1.163</td>
<td>1.090</td>
<td>1.053</td>
<td>1.022</td>
<td>1.049</td>
<td>1.070</td>
<td>1.044</td>
</tr>
<tr>
<td></td>
<td>1.248</td>
<td>0.858</td>
<td>0.956</td>
<td>1.040</td>
<td>0.970</td>
<td>1.001</td>
<td>1.099</td>
<td>1.221</td>
</tr>
<tr>
<td>SDR/Bask</td>
<td>1.005</td>
<td>1.039</td>
<td>1.287</td>
<td>1.371</td>
<td>1.270</td>
<td>1.179</td>
<td>1.169</td>
<td>1.153</td>
</tr>
<tr>
<td></td>
<td>1.158</td>
<td>1.506</td>
<td>1.296</td>
<td>0.935</td>
<td>1.145</td>
<td>1.028</td>
<td>1.501</td>
<td>1.732</td>
</tr>
<tr>
<td>Silver/Bask</td>
<td>0.981</td>
<td>0.649</td>
<td>1.423</td>
<td>0.686</td>
<td>0.967</td>
<td>1.195</td>
<td>0.912</td>
<td>1.043</td>
</tr>
<tr>
<td>Gold/Bask</td>
<td>1.144</td>
<td>0.788</td>
<td>1.010</td>
<td>0.703</td>
<td>1.031</td>
<td>1.317</td>
<td>1.311</td>
<td>1.185</td>
</tr>
<tr>
<td>Currency Pair</td>
<td>6.02.02</td>
<td>6.02.03</td>
<td>6.02.04</td>
<td>6.02.05</td>
<td>6.02.06</td>
<td>6.02.07</td>
<td>6.02.08</td>
<td>6.02.09</td>
</tr>
<tr>
<td>--------------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>EUR/USD</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td><strong>0.260</strong></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>GBP/USD</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td><strong>0.128</strong></td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>SDR/USD</td>
<td>0.839</td>
<td>0.011</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>JPY/USD</td>
<td><strong>0.161</strong></td>
<td><strong>0.148</strong></td>
<td><strong>0.065</strong></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>PLN/USD</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td><strong>0.434</strong></td>
<td><strong>0.277</strong></td>
<td><strong>0.302</strong></td>
</tr>
<tr>
<td>INR/USD</td>
<td>0.000</td>
<td><strong>0.534</strong></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>BRL/USD</td>
<td>0.000</td>
<td><strong>0.128</strong></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td><strong>0.439</strong></td>
<td><strong>0.723</strong></td>
<td><strong>0.698</strong></td>
</tr>
<tr>
<td>AUD/USD</td>
<td>0.000</td>
<td><strong>0.179</strong></td>
<td><strong>0.935</strong></td>
<td><strong>1.000</strong></td>
<td><strong>0.740</strong></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 11.

Optimal basket risk properties: delay $p=0$, Silver and Gold excluded – eq.(10),

<table>
<thead>
<tr>
<th>Last date</th>
<th>6.02.02</th>
<th>6.02.03</th>
<th>6.02.04</th>
<th>6.02.05</th>
<th>6.02.06</th>
<th>6.02.07</th>
<th>6.02.08</th>
<th>6.02.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD/Bask</td>
<td>1.204</td>
<td>1.275</td>
<td>1.558</td>
<td>1.705</td>
<td>1.570</td>
<td>1.384</td>
<td>1.348</td>
<td>1.140</td>
</tr>
<tr>
<td>EUR/Bask</td>
<td>0.832</td>
<td>1.457</td>
<td>1.044</td>
<td>0.949</td>
<td>1.083</td>
<td>0.966</td>
<td>1.463</td>
<td>1.616</td>
</tr>
<tr>
<td>SDR/Bask</td>
<td>1.452</td>
<td>1.370</td>
<td>1.201</td>
<td>1.070</td>
<td>1.103</td>
<td>1.247</td>
<td>1.227</td>
<td>1.064</td>
</tr>
<tr>
<td>Silver/Bask</td>
<td>1.212</td>
<td>1.033</td>
<td>0.921</td>
<td>1.074</td>
<td>0.959</td>
<td>0.949</td>
<td>1.195</td>
<td>1.266</td>
</tr>
<tr>
<td>Gold/Bask</td>
<td>1.223</td>
<td>1.361</td>
<td>1.856</td>
<td>2.051</td>
<td>1.813</td>
<td>1.443</td>
<td>1.395</td>
<td>1.181</td>
</tr>
<tr>
<td>GBP/USD</td>
<td>0.992</td>
<td>1.779</td>
<td>1.181</td>
<td>0.913</td>
<td>1.155</td>
<td>0.993</td>
<td>1.599</td>
<td>1.812</td>
</tr>
<tr>
<td>JPY/USD</td>
<td>0.951</td>
<td>0.854</td>
<td>1.287</td>
<td>0.781</td>
<td>0.956</td>
<td>1.108</td>
<td>0.954</td>
<td>1.076</td>
</tr>
<tr>
<td>BRL/USD</td>
<td>1.116</td>
<td>0.911</td>
<td>0.963</td>
<td>0.784</td>
<td>1.021</td>
<td>1.189</td>
<td>1.351</td>
<td>1.262</td>
</tr>
</tbody>
</table>

Table 12.

Optimal basket risk properties: delay $p=22$, Silver and Gold excluded – eq.(9),

<table>
<thead>
<tr>
<th>Last date</th>
<th>6.02.02</th>
<th>6.02.03</th>
<th>6.02.04</th>
<th>6.02.05</th>
<th>6.02.06</th>
<th>6.02.07</th>
<th>6.02.08</th>
<th>6.02.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD/Bask</td>
<td>1.029</td>
<td>1.023</td>
<td>1.105</td>
<td>1.149</td>
<td>1.085</td>
<td>1.096</td>
<td>1.126</td>
<td>1.044</td>
</tr>
<tr>
<td>EUR/Bask</td>
<td>0.809</td>
<td>1.456</td>
<td>1.117</td>
<td>0.954</td>
<td>1.096</td>
<td>0.948</td>
<td>1.538</td>
<td>1.835</td>
</tr>
<tr>
<td>SDR/Bask</td>
<td>1.070</td>
<td>1.030</td>
<td>1.017</td>
<td>1.016</td>
<td>1.015</td>
<td>1.062</td>
<td>1.087</td>
<td>1.025</td>
</tr>
<tr>
<td>Silver/Bask</td>
<td>1.107</td>
<td>1.011</td>
<td>0.950</td>
<td>1.031</td>
<td>0.984</td>
<td>0.942</td>
<td>1.237</td>
<td>1.451</td>
</tr>
<tr>
<td>Gold/Bask</td>
<td>1.027</td>
<td>1.027</td>
<td>1.163</td>
<td>1.221</td>
<td>1.119</td>
<td>1.119</td>
<td>1.139</td>
<td>1.052</td>
</tr>
<tr>
<td>JPY/USD</td>
<td>0.870</td>
<td>0.765</td>
<td>1.415</td>
<td>0.680</td>
<td>0.981</td>
<td>1.125</td>
<td>1.027</td>
<td>1.240</td>
</tr>
<tr>
<td>BRL/USD</td>
<td>1.014</td>
<td>0.928</td>
<td>1.004</td>
<td>0.696</td>
<td>1.046</td>
<td>1.239</td>
<td>1.476</td>
<td>1.409</td>
</tr>
<tr>
<td>AUD/USD</td>
<td>0.000</td>
<td>0.000</td>
<td><strong>1.000</strong></td>
<td><strong>1.000</strong></td>
<td><strong>1.000</strong></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Table 13.
Optimal basket risk properties: delay \( p = 522 \), Silver and Gold excluded – eq.(8),

<table>
<thead>
<tr>
<th>Last date</th>
<th>6.02.04</th>
<th>6.02.05</th>
<th>6.02.06</th>
<th>6.02.07</th>
<th>6.02.08</th>
<th>6.02.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD/Bask</td>
<td>1.692</td>
<td>2.081</td>
<td>1.849</td>
<td>1.759</td>
<td>1.640</td>
<td>1.326</td>
</tr>
<tr>
<td>EUR/Bask</td>
<td>1.053</td>
<td>0.965</td>
<td>1.056</td>
<td>1.012</td>
<td>1.300</td>
<td>1.524</td>
</tr>
<tr>
<td>SDR/Bask</td>
<td>1.187</td>
<td>1.099</td>
<td>1.149</td>
<td>1.570</td>
<td>1.453</td>
<td>1.150</td>
</tr>
<tr>
<td>Silver/Bask</td>
<td>0.935</td>
<td>1.099</td>
<td>0.934</td>
<td>0.993</td>
<td>1.062</td>
<td>1.195</td>
</tr>
<tr>
<td>Gold/Bask</td>
<td>2.051</td>
<td>2.654</td>
<td>2.241</td>
<td>1.838</td>
<td>1.707</td>
<td>1.417</td>
</tr>
<tr>
<td>BRL/USD</td>
<td>1.187</td>
<td>0.927</td>
<td>1.127</td>
<td>1.040</td>
<td>1.420</td>
<td>1.708</td>
</tr>
<tr>
<td>AUD/USD</td>
<td>1.296</td>
<td>0.807</td>
<td>0.928</td>
<td>1.156</td>
<td>0.851</td>
<td>1.009</td>
</tr>
<tr>
<td>Basket</td>
<td>0.980</td>
<td>0.809</td>
<td>0.993</td>
<td>1.236</td>
<td>1.199</td>
<td>1.185</td>
</tr>
</tbody>
</table>

Table 14.
Optimal basket risk properties: delay \( p = -522 \), Silver and Gold excluded – eq.(9),

<table>
<thead>
<tr>
<th>Last date</th>
<th>6.02.04</th>
<th>6.02.05</th>
<th>6.02.06</th>
<th>6.02.07</th>
<th>6.02.08</th>
<th>6.02.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD/Bask</td>
<td>1.230</td>
<td>1.977</td>
<td>1.740</td>
<td>1.637</td>
<td>1.693</td>
<td>1.216</td>
</tr>
<tr>
<td>EUR/Bask</td>
<td>1.097</td>
<td>0.964</td>
<td>1.057</td>
<td>1.010</td>
<td>1.309</td>
<td>1.291</td>
</tr>
<tr>
<td>SDR/Bask</td>
<td>1.000</td>
<td>1.097</td>
<td>1.134</td>
<td>1.479</td>
<td>1.499</td>
<td>1.073</td>
</tr>
<tr>
<td>Silver/Bask</td>
<td>0.974</td>
<td>1.098</td>
<td>0.936</td>
<td>0.991</td>
<td>1.069</td>
<td>1.012</td>
</tr>
<tr>
<td>Gold/Bask</td>
<td>1.343</td>
<td>2.422</td>
<td>2.039</td>
<td>1.702</td>
<td>1.759</td>
<td>1.281</td>
</tr>
<tr>
<td>Basket</td>
<td>1.021</td>
<td>0.809</td>
<td>0.994</td>
<td>1.234</td>
<td>1.207</td>
<td>1.003</td>
</tr>
</tbody>
</table>

4. CONCLUSIONS

Data presented in Tables 3-14 give evidence that the optimized basket makes possible noticeable reduction of commodity trade risk, when applied instead of typical currencies. It is especially advantageous, when compared to USD, but also to SDR and EURO, during the crisis 2008-2009. In optimization intervals the ratio of the formal risk measures (risk ratio) for these currencies often exceeds the level of 1.5 (50%), and during the crisis 2008-2009 it is mostly larger. The advantage of the optimized basket is evident for large delivery delay (\( p=522 \) days) during the crisis, when the risk ratio reaches huge values of 8.00 and more.

The risk measure calculated for typical homogeneous currencies in application intervals is also mostly higher than that for the optimised basket. The optimized basket were the most effective in 2007, especially for transactions of 2-years delay. During the crisis the risk ratio is
always greater than 1, hence the optimised basket may be recommended for such situations. It concerns mainly the baskets minimizing the price returns $\Delta\Pi$ variance (Tables 3, 5, 10, 12-14).

Optimal basket structure is changeable, in consecutive intervals. The changes are largest in the case of baskets minimizing variance of $\Pi$ - eq.(10). The baskets minimizing the variance of $\Delta\Pi$ are more stable. For $p=1y$ and $2y$ they are based mainly on Gold and Silver. It means that Gold and Silver (used as an instrumental currency) may be recommended to minimize long delay transaction risk. What interesting, the leading currencies (USD, EURO), are rarely included into the basket. In recent years, the currencies attractive for short delay transactions were: JPY, BRL, AUD, Gold and Silver. The most complex baskets are for the 1 month delay (see Table 3). Gold and Silver come very often to the basket, and frequently as the unique currency (making homogeneous basket) – see Tables 4, 5, 7, 8, 9. It is especially visible in Table 9, where the basket efficiency is characterized for 2-year transaction delay. For such transactions, in the last four years Silver was the least risky currency (Tables 8, 9). Silver minimizes also the risk of 1 month delay, but not for price returns $\Delta\Pi$ (see Table 6).

Very interesting results were obtained for the baskets with Silver and Gold excluded. The baskets minimizing $\Delta\Pi$ are very diversified, like these containing Silver and Gold (see Table 10). The dominating currency is BRL, but EURO, GPB, INR and PLN appear too. The basket efficiency is moderately high – particularly when related to SDR and USD. The baskets minimizing $\Delta\Pi$ for large delay are more homogeneous. The leading currencies are mostly BRL and AUD – see Tables 11-14.

Our calculations show that for commodity market USD, GBP are the risky currencies. SDR was moderately advantageous for 1 month delay transactions, since 2000 to 2002. In other years it was 1.5 to twice more risky than optimal basket, both for short and long delay.

References


